Non-linear solving algorithm of X-FEM

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Abstract

In this brief report, some of the algorithmic processes of X-FEM are described focusing on the non-linear solver. Non-linear solving of X-FEM is based on modified Newton-Raphson technique (iterative solving scheme).

1 STEP I: Consider loading steps [loading history]

If large deformations and / or extensive plastic areas are predicted to occur, then divide a loading to several loading steps to speed-up convergence and to increase accuracy as showed to the Figure 1.

2 STEP II: For each loading step, make iterative solution

Consider an iterative solution to take out Residual Forces. X-FEM may use a Newton - Raphson or a modified Newton - Raphson solving scene (Figure 2).

For the first iteration ($i = 0$) load with the Loading Step, for the iteration i ($i > 0$) , load with the residual forces R_{i-1} , until $||R|| \leq \epsilon \cdot ||LS||$ ($\epsilon = 0.1 - 5\%$).

When a Modified Newton Raphson is considered, it is preferable to calculate an elastic $[K]$ for each loading step rather than a unique $[K]$ for all L.S. to

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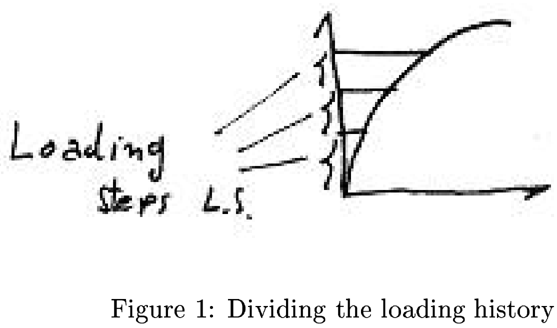

Figure 1: Dividing the loading history to Loading Steps

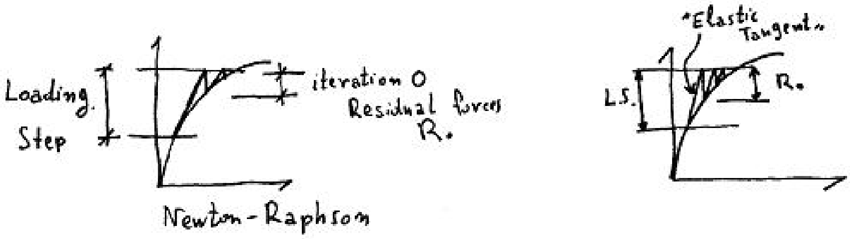

Figure 2: Iterative processes: Newton – Raphson (a) Modified Newton – Raphson (b)

incorporate stiffness variation due to large deformation. In the majority of cases, it is preferable to use the elastic stiffness matrix instead of using the elastoplastic one. On difficult cases of convergence (such as stress softening of soils), only the use of elastic stiffness matrix make convergence possible.

XFEM allows:

- Use of elastoplastic $[Kep]$:
  - calculate $[Kep]$ for each iteration (Original N-R)
  - calculate a $[Kep]$ for each loading step
- Use of elastic $[Kel]$:
  - calculate a unique $[Kel]$ for all loading steps
  - calculate a $[Kel]$ for each loading step

We think that the best solution mode, saving calculation time is: use of $[Kel]$, calculated for each loading step, unless no large deformation: in that case use the unique $[Kel]$.

3 STEP III: Elastoplastic stress calculation

The process of elastoplastic stress calculation is described by: "Hinton, Owen – Finite Element Method in Plasticity". Some improvements are made to these algorithms.

For each iteration i:

- Calculate deformation $[u]_i$ by the standard FEM solution $[K] \cdot [u] = [F] + [R]$
- For each Gauss point of each element, calculate a strain tensor increase: $[d\epsilon]_i$, by differentiation ($\epsilon = [B] \cdot [u]^T$, $[B]$ the derivate of the shape functions)
- Calculate an elastoplastic or plastic stress increase $[d\sigma]$ as represented to the Figure 3.

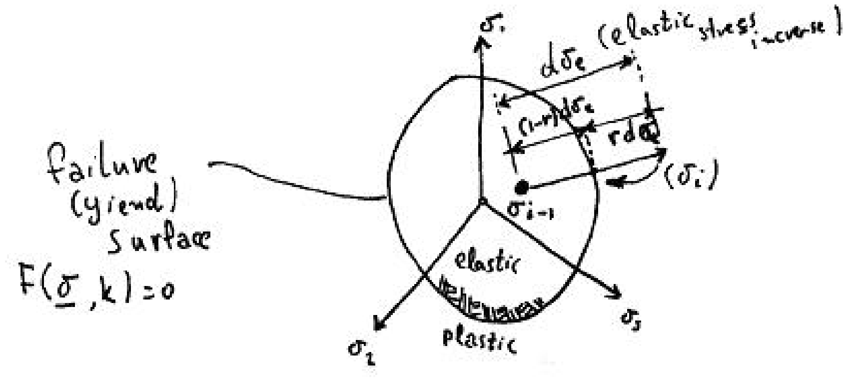

Figure 3: Factoring strain increment ϵ with r to divide elastic and elastoplastic strains

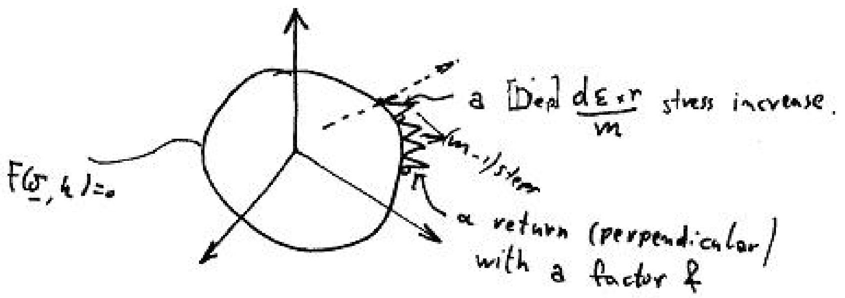

Figure 4: Iterative process to calculate the elastoplastic stress increment

r is a scalar factor, scale $[d\sigma e]$ with $(1-r)$. The rest $r[d\epsilon]$ strain increase concerns to plastic strains. If $i-1$ iteration was "plastic", then set $r=1$.

For the $r[d\epsilon]$ plastic strain increase, divide $r[d\epsilon]$ to finer increases such as:

$$r[d\epsilon]/m$$

I use a value of $m = 10$ to 12 (e.g. $9-11$ steps). For each $r[d\epsilon]/m$, calculate a stress increase:

$$[d\sigma_{ep}] = [D_{ep}] * r[d\epsilon]/m$$

Now, check if $[\sigma] + [d\sigma_{ep}]$ lies on the failure surface. If it doesn't, scale $[\sigma] + [d\sigma_{ep}]$ with the elastoplastic factor f, so it does.

I prefer to scale the divergent stresses rather than the full stress tensor, to allow for perpendicular return to the failure surface. e.g.:

$$[p + f\tilde{\sigma}_x, p + f\tilde{\sigma}_y, p + f\tilde{\sigma}_z, \tau_{xy}, \tau_{yz}, \tau_{zx}] < \tilde{\sigma}_{x,y,z} = \sigma_{x,y,z} - p,\ p = \frac{I_1}{3} >$$